Differential Calculus

How to Approach Limits in Calculus

QUESTION [7] 2.PE.23

Find the limit or explain why it does not exist,

$$\lim_{x\to 0}\frac{8x}{3\sin x - x}.$$

ANSWER

$$4$$

First plug $x = 0$ in

$$\lim_{x\to 0}\frac{8x}{3\sin x - x} = \frac{8(0)}{3\sin(0) - (0)} = \frac{0}{0} = \text{indeterminant.}$$

Since the limit is in indeterminate form, $\frac{0}{0}$, we could use L'Hospital's Rule if allowed. But we are not allowed yet to do this. So, we must manipulate the function.

The question is: What are our options? We begin by building an algorithmic list:

[1] Is the limit in indeterminant form?
[2] Can I factor?

$$\lim_{x\to 0}\frac{8x}{3\sin x - x} = \lim_{x\to 0}\frac{x(8)}{x\left(\frac{3\sin x}{x} - 1\right)} = \lim_{x\to 0}\frac{8}{\frac{3\sin x}{x} - 1}$$

Factoring is the correct approach. Now, we must use limit laws for ratio and then sum and difference and then look at a preexisting formula.

$$= \frac{\lim_{x\to 0} 8}{\lim_{x\to 0}\left(\frac{3\sin x}{x} - 1\right)} = \frac{\lim_{x\to 0} 8}{\lim_{x\to 0}\left(\frac{3\sin x}{x}\right) - \lim_{x\to 0}(1)} = \frac{8}{\lim_{x\to 0}\left(\frac{3\sin x}{x}\right) - 1}$$

Identify

Theorem 7 from Thomas' Calculus

$$\lim_{x\to 0}\frac{\sin x}{x} = 1.$$

[RP] Prove (or show) Theorem 7 is correct.

Thus,

$$= \frac{8}{\lim\limits_{x\to 0}\left(\frac{3\sin x}{x}\right) - 1} = \frac{8}{3\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right) - 1} = \frac{8}{3-1} = \frac{8}{2} = 4.$$

EXAM (for max credit) Find the limit or explain why it does not exist,

$$\lim_{x\to 0}\frac{8x}{3\sin x - x}.$$

Student Exam Solution

First

$$\lim_{x\to 0}\frac{8x}{3\sin x - x} = \frac{8(0)}{3\sin(0) - (0)} = \frac{0}{0} = \text{indeterminant}.$$

Second (identify approach)

$$\lim_{x\to 0}\frac{8x}{3\sin x - x} = \lim_{x\to 0}\frac{x(8)}{x\left(\frac{3\sin x}{x} - 1\right)} = \lim_{x\to 0}\frac{8}{\frac{3\sin x}{x} - 1}$$

Citation for this solution (English Essay) By **Theorem 6**, (worth 25-50% of the grade)

$$\text{T6: } \lim_{x\to 0}\frac{\sin x}{x} = 1.$$

Thus,

$$= \frac{8}{\lim\limits_{x\to 0}\left(\frac{3\sin x}{x}\right) - 1} = \frac{8}{3\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right) - 1} = \frac{8}{3(1)-1} = \frac{8}{2} = 4.$$

Student Solution

$$\lim_{x\to 0}\frac{8x}{3\sin x - x} = \frac{8(0)}{3\sin(0) - (0)} = \frac{0}{0} = \text{indeterminant}.$$

Second (identify approach)

$$\lim_{x\to 0}\frac{8x}{3\sin x - x} = \lim_{x\to 0}\frac{x(8)}{x\left(\frac{3\sin x}{x} - 1\right)} = \lim_{x\to 0}\frac{8}{\frac{3\sin x}{x} - 1}, \qquad \text{T7: } \lim_{x\to 0}\frac{\sin x}{x} = 1.$$

Thus,

$$= \frac{8}{\lim\limits_{x\to 0}\left(\frac{3\sin x}{x}\right) - 1} = \frac{8}{3\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right) - 1} = \frac{8}{3(1)-1} = \frac{8}{2} = 4.$$

